

NON-GAUSSIANITY TEST FOR DISCRIMINATING GRAVITATIONAL WAVE BACKGROUNDS AROUND 0.1-1HZ

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ABSTRACT

We propose a non-Gaussianity test for gravitational wave backgrounds by combining data streams of multiple detectors. This simple method allows us to check whether a detected background is "smooth" enough to be consistent with an inflation-type background, or is contaminated by individually undetectable weak burst signals. The proposed test would be quite useful for the Big Bang Observer or DECIGO whose primary target is a background from inflation at 0.1-1Hz where gravitational wave bursts from supernovae of population III stars might become a troublesome foreground.

Subject headings: gravitational waves— early universe — supernovae: general

1. INTRODUCTION

Since gravitational interaction is very weak, a gravitational wave (GW) background can serve as an invaluable fossil from early universe with almost no scattering and absorption during its propagation (Maggiore 2000). By analyzing the background, we might obtain crucial information to understand physics at extremely high energy scale, *e.g.* the inflation process. However, overlap of multiple signals in data streams of detectors would become a basic aspect of GW astronomy, especially in the low frequency regime where number and variety of astrophysical sources would increase. This is partly because GW detectors have omni-directional sensitivity. Therefore, for detecting a background from early universe, it is essential to disentangle diversified signals contained in observed data. For example, we need to detect and subtract individual astrophysical signals such as chirping binaries, from data streams (see *e.g.* Arnaud et al. 2007; Harms et al. 2008 for recent progress).

The primary target for the Big Bang Observer (BBO, Phinney et al. 2003) and the DECihertz Interferometer Gravitational wave Observatory (DECIGO, Seto et al. 2001) is a background from inflation in the 0.1-1Hz band where a relatively deep window of GWs has been expected to be opened. However, it was recently pointed out that burst-like GWs produced at supernova (SN) explosions of population-III (popIII) stars might become a problem for detecting an inflation background around 0.1-1Hz (Buonanno et al. 2004; Sandick et al. 2006; Suwa et al. 2007). While a burst signal with a large amplitude might be handled in data analysis, a potential background composed by weak undetectable bursts would be a formidable obstacle for adequately identifying an inflation background. Furthermore, in contrast to regular waveforms (*e.g.* from binaries), it is often difficult to accurately model waveforms for burst signals and/or to predict their characteristic amplitudes beforehand. To deal with such a situation, we certainly need to study GW backgrounds with efficient quantification methods beyond the traditional simple measure Ω_{GW} , the energy density of backgrounds (see also Finn et al. 1999; Drasco & Flanagan 2002). In this paper we propose a non-Gaussianity test for GW backgrounds in 0.1-1Hz band, and discuss its prospects with BBO, setting GWs from the popIII SNe as our fiducial burst model.

This paper is organized as follows: in §2 we briefly describe data streams of BBO with summaries for notations. The basic idea behind our non-Gaussianity test is presented in §3. In §4 we make a statistical evaluation for our method.

2. DATA STREAMS OF BBO

For BBO, two LISA-type units (armlength $L = 5 \times 10^4$ km) would be used to form a David-star-like configuration. From each unit we can obtain three data streams (A, E, T) and (A', E', T') (Prince et al. 2002; Corbin & Cornish 2005; Seto 2006). Among these six data, T and T' modes are less sensitive to GWs, and we neglect them hereafter. The data streams A, E, A' and E' can be effectively regarded as responses of simple L-shaped detectors around their optimal frequency $f_{opt} \sim 0.3$ Hz with bandwidth $\delta f \sim f_{opt}$ (Seto 2006). In this paper we only deal with quantities made from the following two pairs; $A - A'$ or $E - E'$. The orientation of the arms of the former pair is aligned on the common detector plane of the two units, but the latter is misaligned by 45° on the plane.

We model the data streams $s_X(t)$ ($X = A, E$ and A') in terms of GW signal H_X and detector noises n_X as $s_X(t) = H_X(t) + n_X(t)$. For analyzing the noises n_X , it is advantageous to work in the Fourier space. To clarify our main points, we discuss signal analysis in the optimal band with neglecting details of frequency dependence (*e.g.* replacing an integral $\int(\cdots)df$ with a product $(\cdots)_{f_{opt}} \times \delta f$). In practice, this situation is approximately realized by applying a band-pass filter. In order to take Fourier transformation, we decompose the data streams (total duration T_{obs}) into short segments of a given period $T_{seg} (\gtrsim f_{opt}^{-1})$, and assign a label $M (= 1, \cdots, T_{obs}/T_{seg})$ for each segment with chronological order. Then we calculate

$$s_{XM}(f) = \int_{(M-1)T_{seg}}^{MT_{seg}} s_X(t) e^{2\pi i f t} dt = H_{XM}(f) + n_{XM}(f). \quad (1)$$

The number of relevant Fourier modes in a segment is $\sim T_{seg}\delta f \sim T_{seg}f_{opt}$. In the Fourier transformations above, we implicitly assumed to apply an adequate time window function to suppress leakage of underlying frequency components to nearby modes due to finiteness of T_{seg} .

Hereafter, we only used the Fourier transformed quantities, assuming that the detector noises n_X are stationary, Gaussian, and have identical spectrum $S_N(f)$. The assumption of Gaussian noises is not crucial for our method. For the relevant pairs $(X, Y) = (A, A')$ or (E, A') , we also assume that their noises are independent (Phinney et al. 2003) with

$$\langle n_{XM}(f)n_{YL}(f')^* \rangle \sim \frac{1}{2}\delta_{ML}\delta_{XY}\delta_{ff'}T_{seg}S_N(f), \quad (2)$$

where the notation $\langle \dots \rangle$ represents to take an ensemble average.

Next we discuss detectors' responses to incoming GWs. We expand the metric perturbation due to a GW background at a time t and position \mathbf{x} by

$$h_{ij}(t, \mathbf{x}) = \sum_{P=+, \times} \sum_{\mathbf{f}} e^{2\pi i(\mathbf{f}\mathbf{x}+tf)} h_{In}(P, \mathbf{f}) e_{ij}^P + \text{c.c.} \quad (3)$$

(c.c.: complex conjugate) with three-dimensional wave vectors \mathbf{f} , polarization bases e_{ij}^P and a definition $f \equiv |\mathbf{f}|$. Response of a detector X to an incident GW is characterized by the beam pattern function $F_X(\mathbf{f}, P)$. In this paper, unless otherwise stated, we study simple L-shaped interferometers with the long-wave approximation. The explicit form of the function F_X is presented in the literature (Flanagan 1994; Allen & Romano 1997). For the background above, the signal $H_{XM}(f)$ in eq.(1) is expressed as

$$H_{XM}(f) \simeq \sum_{P=+, \times} \sum_{\mathbf{f} \in B_f} \exp[2\pi i(\mathbf{f} \cdot \mathbf{x}_X + ft_M)] h_{In}(\mathbf{f}, P) T_{seg} F_X(\mathbf{f}, P) \quad (4)$$

with a shell-like three-dimensional frequency region B_f corresponding to the observed frequency f . Then we have

$$\langle H_{XM}(f)H_{YM}(f)^* \rangle = \frac{8\pi}{5} S_{GW,I}(f) \gamma_{XY} T_{seg}, \quad (5)$$

where the spectrum $S_{GW}(f)$ for the background has dimension $[\text{Hz}^{-1}]$ as for the detector noise spectrum $S_N(f)$, and is written with the normalized energy density Ω_{GW} by $S_{GW}(f) = \frac{3H_0^2}{32\pi^3 G} f^{-3} \Omega_{GW}(f)$ ($H_0 \simeq 70 \text{ km/sec/Mpc}$: the Hubble parameter). The overlap function $\gamma_{XY}(f)$ characterizes magnitude of common responses of two detectors (X, Y) to isotropic and unpolarized backgrounds (Flanagan 1994; Allen & Romano 1997). It is defined by

$$\gamma_{XY}(f) \equiv \frac{5}{8\pi} \int_{\text{Sphere}} d\mathbf{n} (F_X^+ F_Y^{+*} + F_X^\times F_Y^{\times*}) e^{2\pi i f \mathbf{n} \cdot (\mathbf{x}_a - \mathbf{x}_b)}. \quad (6)$$

We have $\gamma_{EA'} = 0$ due to geometrical symmetry and also $\gamma_{AA'} \sim 1$ around the optimal band $f \sim 0.3 \text{ Hz}$ of BBO (Seto 2006).

3. NON-GAUSSIANITY TEST

If the signals $H_{XM}(f)$ in eq.(4) are made from superpositions of vast number of incoherent waves, they can be regarded as Gaussian variables from the central limit theorem. As a result, they are characterized only by second moments, and we have

$$\langle (H_{XM}(f)H_{YM}(f)^*)^2 \rangle = 2 \langle H_{XM}(f)H_{YM}(f)^* \rangle^2 \propto \gamma_{XY}^2 \quad (7)$$

from the properties such as $\langle H_{XM}(f)H_{XM}(f) \rangle = 0$.

For a "coarse background" made by a relatively small number of freedom (*e.g.* popcorn noise due to supernova bursts), the responses of detectors would be deviate from Gaussian. Therefore, using the BBO pair $E-A'$ with the overlap function $\gamma_{EA'} = 0$, we can check granularity of an isotropic background through the quantity $\langle (H_{EM}(f)H_{A'M}(f)^*)^2 \rangle$ that should vanish for a Gaussian background *e.g.* that generated at inflation. This is a key point in this paper. For detectors with a finite overlap $\gamma_{XY} \neq 0$, we can generalize this method by introducing combinations such as $\langle (H_{XM}(f)H_{YM}(f)^*)^2 \rangle - 2 \langle H_{XM}(f)H_{YM}(f)^* \rangle^2$ to subtract Gaussian component (similar to the definition of the Kurtosis parameter κ_4 for standard characterization of non-Gaussianity (see *e.g.* Racine & Cutler 2007)). We do not pursue this direction further. But the underlying approach proposed in this paper would be applicable to a network of ground based detectors. For general detector configurations such as the LIGO-VIRGO pair, we need the subtraction scheme described above.

For statistical analysis with BBO, we introduce the following two quantities made from the two pairs $A - A'$ and $E - A'$ respectively:

$$C_2 \equiv \sum_M \sum_{f \in \delta f} s_{AM}(f) s_{A'M}(f)^*, \quad C_4 \equiv \sum_M \sum_{f \in \delta f} (s_{EM}(f) s_{A'M}(f)^*)^2 \quad (8)$$

Here, the second summations $\sum_{f \in \delta f}$ are for the Fourier modes (total number: $T_{seg}\delta f$) within a segment, and the first ones \sum_M are for the segment label $M = (1, \dots, T_{obs}/T_{seg})$. The combination C_2 is used for traditional correlation

analysis to measure Ω_{GW} , while C_4 is our new probe for non-Gaussianity of a GW background. We evaluate their signal-to-noise ratios in the next section.

In this paper we have set GWs from popIII SNe as our fiducial burst model. Here we comment on other models. Recently, several cosmological scenarios were proposed to produce intrinsically non-Gaussian GWs background in early universe. With a typical cosmological mechanism (*e.g.* through preheating phases), GW background is generated by causal processes, when wavelength were comparable to or smaller than horizon size in order of magnitude sense (see *e.g.* Easther & Lim 2006). Therefore, even if the generated GWs have a correlation structure, we have a large number ($\gtrsim (fH_0^{-1})^2 \sim 10^{36}(f/1\text{Hz})^2$) of independent emission regions for GWs currently observed at frequency f , and it would be difficult to directly probe the intrinsic non-Gaussianity for these typical models with BBO, due to the central limit theorem. But a background made by sparse cosmological events, such as GW bursts from cusps of cosmic strings might be an interesting target (Damour & Vilenkin 2005).

4. BURST BACKGROUND

4.1. Derivation of Formulas

In this section we analyze a GW background made by a superposition of burst events from single-species sources that have an event rate R and a characteristic duration time T_d for GW signal in the optimal band. We start our discussion in a somewhat general way, and derive useful expressions for C_2 and C_4 . Our fiducial model (GWs popIII SNe) will be examined later.

We assume a smooth spectral profile $\mathcal{A}(f)^2$ ($\mathcal{A} > 0$) for the GW emission throughout a burst event, and do not deal with regular waveforms with sharp frequency structures (*e.g.* monochromatic waves or chirping waves). For simplicity, the emitted wave pattern observed at the Earth is assumed to have an axisymmetric profile described by

$$(h_+, h_\times) \propto (\alpha_+(I), \alpha_\times(I)) \quad (9)$$

(I : inclination angle, $\alpha_{+,\times}$: complex number) in the coordinate system that is apparently symmetric to the geometry of the source. We also impose a normalization condition; $\frac{1}{2} \int_0^\pi (|\alpha_+|^2 + |\alpha_\times|^2) \sin I dI = 1$.

For our probes C_2 and C_4 , we need to evaluate the second- and forth-order moments of the responses H_{XM} of two detectors induced by a burst event. Each response H_{XM} depends on the geometry of the source relative to the detector. The geometry is characterized by four angular parameters; the direction (θ, ϕ) of the source on the sky and the orientation (I, ψ) of its axis, and we have

$$H_{XM} \propto \mathcal{A}(F_X^+(\theta, \phi, \psi)\alpha_+(I) + F_X^\times(\theta, \phi, \psi)\alpha_\times(I)). \quad (10)$$

Since these four geometric parameters are randomly distributed for extra-Galactic events, we define an averaging operator $[\dots]_{an}$ with respect the direction and orientation of sources, and obtain

$$[H_{XM}H_{YM}^*]_{an} = \frac{\mathcal{A}^2}{5} \gamma_{XY}. \quad (11)$$

We define the ratio $Q \equiv ([H_{AM}H_{A'M}^*]^2)_{an}/([H_{AM}H_{A'M}^*]_{an})^2$ for quantitative evaluation of the probe C_4 . Its numerator is explicitly given as

$$([H_{AM}H_{A'M}^*]^2)_{an} = \frac{\mathcal{A}^4}{630} \int_0^\pi (9\{|a_+|^4 + |a_\times|^4\} - 34|a_+a_\times|^2 + 52\text{Re}[(a_+^*a_\times)^2]) \sin I dI. \quad (12)$$

Unless a weird cancellation occurs, the ratio Q becomes order of unity. At the end of this subsection, we will explicitly demonstrate this for our fiducial popIII SNe model.

With a relation $\gamma_{XX} = 1$ for a self correlation, eq.(11) shows that the angular average of the response function is $1/\sqrt{5}$. Therefore, the characteristic signal-to-noise ratio SNR_{Bst} for individual burst with a single detector is evaluated as (see *e.g.* Segalis & Ori 2001; Sago et al. 2004)

$$SNR_{Bst} \sim \frac{2\langle \mathcal{A}^2 \rangle^{1/2} (\delta f)^{1/2}}{5^{1/2} S_N^{1/2}}. \quad (13)$$

Next we analyze the GW background formed by incoherent superposition of the bursts analyzed above, assuming that the typical signal-to-noise ratio SNR_{Bst} is not larger than $O(1)$. As the expected number of events in a segment is RT_{seg} , we get the background spectrum

$$S_{GW,Bst} = \frac{R\langle \mathcal{A}^2 \rangle}{8\pi} \quad (14)$$

from expression (5) and the corresponding mean magnitude $\langle H_{XM}H_{YM}^* \rangle = RT_{seg} \times \langle \mathcal{A}^2 \rangle \gamma_{XY}/5$. Now we evaluate the signal-to-noise ratio $SNR_{Bst,C2}$ for the quantity C_2 under the condition $T_{seg} \gg T_d$. If a segment M contains a burst event, its averaged contribution to C_2 is $\langle \mathcal{A}^2 \rangle \gamma_{XY} T_{seg} \delta f/5$. After incoherent superposition of $T_{obs}R(\gg 1)$ events during observation time T_{obs} , we obtain the expectation value (signal strength) $\langle C_2 \rangle = \langle \mathcal{A}^2 \rangle \gamma_{XY} T_{seg} \delta f/5 \times (T_{obs}R)$. Meanwhile, assuming independence of detector noises and the condition $S_N(f_{opt}) > S_{GW}(f_{opt})$, the rms fluctuations for the product $s_{AM}s_{A'M}^*$ are given by the detector noise spectrum as $2^{-1/2}2^{-1}S_N T_{seg}$. Here the additional factor

$2^{-1/2}$ is associated with the projection operation of data to the expected phase direction (usually onto the real axis) in the complex plane (Seto 2006). Taking into account the total number of Fourier modes $(T_{obs}/T_{seg}) \times (T_{seg}\delta f) = T_{obs}\delta f$, we obtain the rms fluctuations for C_2 as $2^{-3/2}S_N T_{seg}(T_{obs}\delta f)^{1/2}$, and its signal-to-noise ratio is given by

$$SNR_{Bst,C2} = \frac{2}{5} \frac{R \langle \mathcal{A}^2 \rangle}{S_N} (2T_{obs}\delta f)^{1/2}. \quad (15)$$

We can derive the same result for $T_{seg} \lesssim T_d$. After replacing the product $\times \delta f$ with a frequency integral $\int df$ and using the spectrum $R \langle \mathcal{A}^2 \rangle = 8\pi S_{GW,Bst}$, the square value of this expression exactly matches with the standard formula for correlation analysis given in the literature (Flanagan 1994; Allen & Romano 1997).

In the same manner, we can derive $\langle C_4 \rangle = \langle \mathcal{A}^4 \rangle Q R T_{seg} T_{obs} \delta f / 25$ and obtain its signal-to-noise ratio as

$$SNR_{Bst,C4} = \frac{2^{3/2} \langle \mathcal{A}^4 \rangle |Q| R (T_{obs}\delta f)^{1/2}}{25 T_{seg} S_N^2} \quad (\text{for } T_{seg} \gg T_d) \quad (16)$$

with an explicit dependence on T_{seg} in contrast to C_2 . Note that, here, independence of detector noises is an important requirement, but their Gaussianity is not essential.

For $T_{seg} \lesssim T_d$, we get the expectation value $\langle C_4 \rangle = \langle \mathcal{A}^4 \rangle Q R T_{seg}^2 T_{obs} T_d^{-1} \delta f / 25$ with its signal-to-noise ratio

$$SNR_{Bst,C4} = \frac{2^{3/2} \langle \mathcal{A}^4 \rangle |Q| R (T_{obs}\delta f)^{1/2}}{25 T_d S_N^2} \quad (\text{for } T_{seg} \lesssim T_d). \quad (17)$$

Eqs.(16) and (17) are re-expressed as $SNR_{Bst,C4} = \frac{U}{4\delta f \max\{T_d, T_{seg}\}} SNR_{Bst,C2} \cdot SNR_{Bst}^2$ with $U \equiv |Q| \langle \mathcal{A}^4 \rangle / \langle \mathcal{A}^2 \rangle^2 = O(1)$. With increasing the segment length T_{seg} from its minimum $\sim f_{opt}^{-1}$, we have a transition from $\langle C_4 \rangle \propto T_{seg}^2$ to $\langle C_4 \rangle \propto T_{seg}^1$ at the point $T_{seg} \sim T_d$ where the signal-to-noise ratio $SNR_{Bst,C4}$ also starts to decrease from a constant given in eq.(17) due to dilution of power. Therefore, if the signal C_4 is detectable at $T_{seg} \sim f_{opt}^{-1}$, we can estimate the duration time T_d by identifying the transition. In the following, we set $T_{seg} \sim f_{opt}^{-1}$ that will provide us with the maximum value of $SNR_{Bst,C4}$ for a burst background.

Now we focus our discussion on a burst model with $T_d \sim f_{opt}^{-1}$ (e.g. for our fiducial popIII SNe model at 0.1-1Hz). In this case, we have $SNR_{Bst,C4} = U \cdot SNR_{Bst,C2} \cdot SNR_{Bst}^2 / 4$, and this relation is very suggestive. Even if individual burst has signal-to-noise ratio SNR_{Bst} less than detection threshold, there is an amplification factor $SNR_{Bst,C2} (\propto T_{obs}^{1/2})$ that increases with observational time T_{obs} and enable us to statistically study the bursts.

With the parameters related to the bursts, we obtain $SNR_{Bst,C4} = (S_{GW,Bst})^2 U / R \times 64\pi^2 (2T_{obs}\delta f)^{1/2} \delta f / (25 S_N^2)$. For a fixed background level $S_{GW,Bst} (\propto R \langle \mathcal{A}^2 \rangle)$, the signal-to-noise ratio $SNR_{Bst,C4}$ decreases for a higher event rate R (corresponding to a smaller amplitude $\langle \mathcal{A}^2 \rangle$). This is reasonable, considering that the background would become more Gaussian-like. If the bursts events are supposed to be the dominant sources of the total GW background around f_{opt} and both $\langle C_2 \rangle \propto R \langle \mathcal{A}^2 \rangle$ and $\langle C_4 \rangle \propto R \langle \mathcal{A}^2 \rangle^2 U / T_d$ are measured, we can roughly estimate the event rate R and the amplitude $\langle \mathcal{A}^2 \rangle$ separately, assuming $U = O(1)$. In addition to the estimated duration T_d , these will be basic information to disclose the nature of the burst sources.

As we commented earlier, the ratio $Q \equiv [(H_{EM} H_{A'M}^*)^2]_{an} / [(H_{AM} H_{A'M}^*)^2]_{an}$ becomes order of unity for typical burst waveforms. Here we demonstrate this for our fiducial model: burst GWs from popIII SNe. In the band around 0.1-1Hz, the emitted waves are dominated by memory effects caused by anisotropic neutrino emissions at supernova explosions (Buonanno et al. 2004), and we put the GW waveforms as $(\alpha_+, \alpha_-) \propto \{\sin^2 I (1 + \frac{3}{5}q - \frac{1}{5}q \sin^2 I), 0\}$ (Epstein 1978). The parameter $q (\lesssim O(1))$ characterizes asphericity of the emission with $q > 0$ for a polar enhancement and $q < 0$ for an equatorial enhancement. Compared with the standard expression, we omitted an overall factor ($\propto q$) that can be absorbed into the power \mathcal{A} . For this model the ratio is given as $Q = 0.99$ ($q = 1$), $Q = 1.02$ ($q = 0$) and $Q = 1.10$ ($q = -1$). We have limited our non-Gaussianity test only with the fixed pair $E - A'$. By combining results from geometrically different pairs, we can in principle, extract polarization information for burst signals.

4.2. Prospects around 0.1-1Hz

In this subsection we specifically discuss prospects of our non-Gaussianity test for popIII SNe GW background with BBO. Since the anisotropies of neutrino emissions from popIII SNe or the formation rate of popIII stars are poorly understood, the amplitude of the popIII background is currently quite uncertain. With a parameter set for popIII SNe (redshift $z \sim 15$, emitted neutrino energy $E_\nu \sim 10^{55}$ erg with mean isotropy $\langle q \rangle \sim 0.03$ and event rate $R \sim 0.01 \text{sec}^{-1}$) extracted from Buonanno et al. (2004), we have $\Omega_{GW} \sim 4 \times 10^{-16}$ at $f \sim 0.3 \text{Hz}$. Meanwhile, for bursts characterized by the background level Ω_{GW} and their rate R , BBO has a sensitivity corresponding to

$$SNR_{Bst} \sim 0.6 \left(\frac{\Omega_{GW}}{4 \times 10^{-16}} \right)^{1/2} \left(\frac{R}{0.01 \text{sec}^{-1}} \right)^{-1/2} \quad (18)$$

for individual bursts, and

$$SNR_{Bst,C2} \sim 80 \left(\frac{\Omega_{GW}}{4 \times 10^{-16}} \right) \left(\frac{T_{obs}}{10 \text{yr}} \right)^{1/2}, \quad (19)$$

$$SNR_{Bst,C4} \sim 10 \left(\frac{\Omega_{GW}}{4 \times 10^{-16}} \right)^2 \left(\frac{R}{0.01 \text{sec}^{-1}} \right)^{-1} \left(\frac{T_{obs}}{10 \text{yr}} \right)^{1/2} \quad (20)$$

for the background with $T_{seg} \cdot \delta f \sim 1$, $Q \sim 1$ and $U \sim 1$. Therefore, while identification of each burst might be difficult with small SNR_{Bst} , our method has potential to discriminate whether a background once detected is smooth enough and consistent with inflation origin.

An interesting question related to our non-Gaussianity test is whether we can separate smooth and burst contributions for the total energy spectrum Ω_{GW} . To estimate the latter component, we need the combination $R \langle \mathcal{A}^2 \rangle$. With our approach based on a forth-order moment, we can obtain a different combination $RQ \langle \mathcal{A}^4 \rangle = (R \langle \mathcal{A}^2 \rangle)^2 \times U/R$. If the burst rate R is independently estimated *e.g.* with optical observation of popIII SNe, we can roughly estimate the burst component $\propto R \langle \mathcal{A}^2 \rangle$ in the total spectrum Ω_{GW} by introducing a model parameter $U = O(1)$.

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